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DECEMBER 1967

CONSIDERATIONS OF APPLYING CONTINUOUS THRUST DURING SYNERGETIC PLANE CHANGING

F. S. Nyland

PREPARED FOR:
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PREFACE

This Memorandum outlines one approach toward solving heating problems encountered by a winged spacecraft performing a synergetic plane change. Previous analyses of this maneuver for changing the orbital plane of a space vehicle have shown that heating problems are severe under a variety of conditions. In this Memorandum, one approach is suggested for reducing heating rates and temperatures by using on-board propulsion to increase lift and counter the drag.

The results of this analysis should be of interest to planners and trajectory and propulsion specialists, as well as to those concerned with flight in space and the upper reaches of the earth's atmosphere.

SUMMARY

This Memorandum outlines an approach toward solving heating problems encountered by winged spacecraft performing a synergetic plane change. Severe heating may occur during the atmospheric portion of flight when bank angles are large, and one method of alleviating some of the heat load is to use power so as to maintain the vehicle at a high altitude. From the closed-form solutions to the equations of motion presented here, it was found that structural temperatures of lifting vehicles can be lowered 150°F to 500°F, compared to the temperatures encountered along trajectories where turning is accomplished during a pure glide phase. The use of propulsion during a turn may permit the use of highly banked flight attitudes which tend to minimize energy expenditures in such maneuvers. Another benefit of cruising is a reduced normal acceleration.

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SYMBOLS

A = aerodynamic reference area

C_D = drag coefficient

C_L = lift coefficient

C_T = thrust fraction ($T = C_T q$)

D = drag force

$f(u)$ = function of velocity (used in heating equations)

g = acceleration due to gravity

H_s = heat at stagnation point

h = altitude

I_{sp} = specific impulse of propulsion system

k = $T(\cos \delta)/D$

L = lift force

Q = total heat input

q = aerodynamic pressure

R_o = earth radius

r_n = nose radius

s = path length

T = thrust force

t = time

u = vehicle velocity

u_o = orbit velocity at reference altitude = $\sqrt{(R_o + h)g}$

W = weight

α = angle of attack

β = inverse scale height

γ = flight-path angle

δ = thrust cant angle

ε = emittance

ρ = atmospheric density

ρ_o = sea-level atmospheric density

φ = bank angle

ω = heading-change angle

I. INTRODUCTION

In a recent discussion of heating constraints,(1) it was pointed out that heating loads on maneuvering spacecraft with wings can be decreased during a synergetic plane change by utilization of deflected engine thrust, if the aerodynamic turn is made with power on. The basic effect of such a scheme is to augment lift with a component of thrust, thus increasing the cruising altitude over what it would be during a pure gliding turn. This Memorandum is devoted to determining closed-form trajectory equations for the powered-flight portion of a synergetic plane change and to exploring the reduced heating rates and temperatures that might result.

The synergetic plane change treated in earlier studies consisted of a deorbit phase, a glide phase during which the vehicle heading was changed, and finally, a powered phase to return the vehicle to a new orbit.(2-5) In this discussion, we shall consider use of propulsion shortly after entry into the atmosphere to cancel drag and provide additional lift so that much of the heading change would be accomplished during a cruise phase at constant vehicle velocity. At the end of the cruise phase, the thrust of the engine would be increased to accelerate the vehicle into a new orbit. This maneuver is illustrated in Fig. 1. The advantage of this approach over trajectories studied previously is that heating rates and vehicle temperatures could be decreased. Trajectories that include changing planes imply the use of a variable-thrust engine which may make the operation more complicated. A constant-thrust-engine approach, which may lead to equivalent performance, is also possible; however, this has not been considered here.

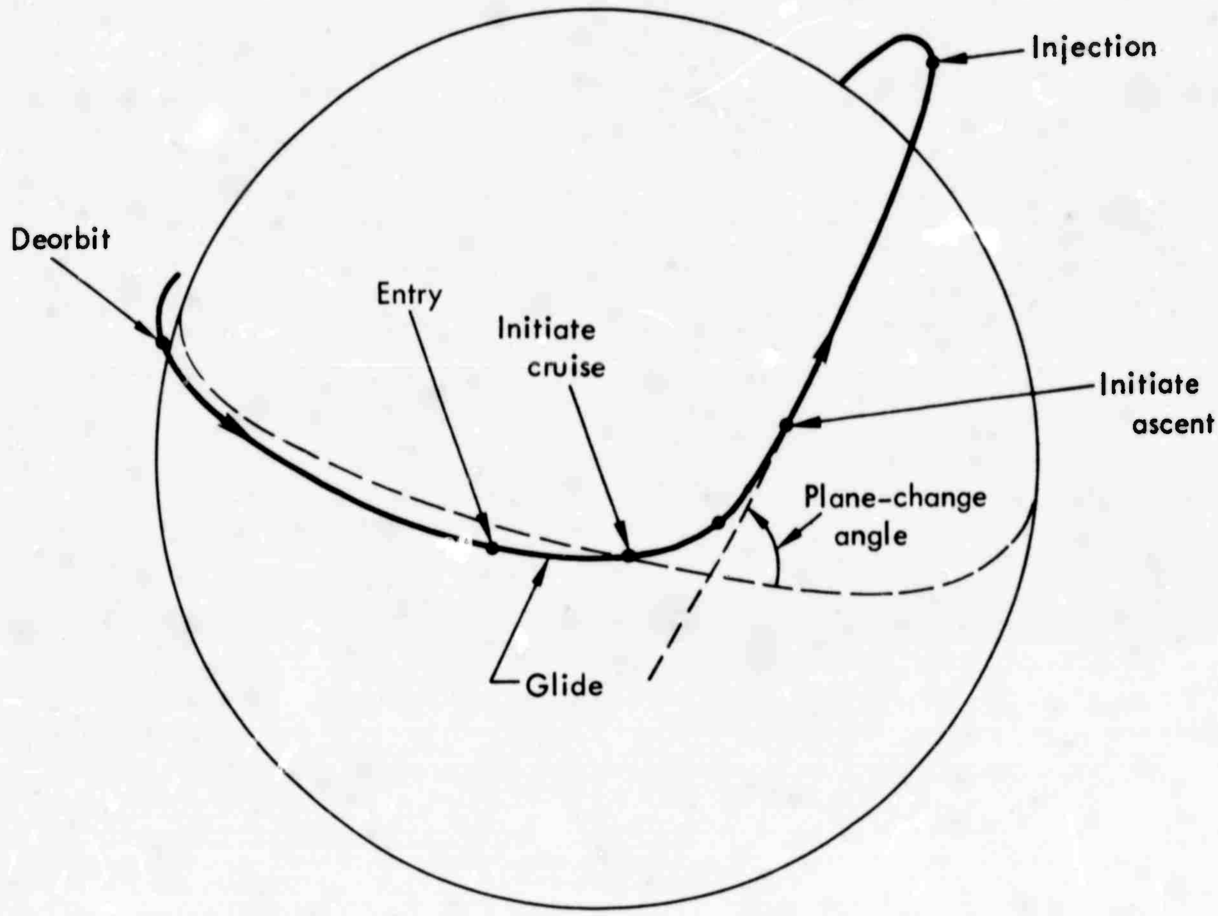

Fig. 1—Synergetic turn with cruise

II. ANALYSIS

ASSUMPTIONS

As has been noted in previous analyses,[6] it is possible to make certain simplifying assumptions for finding analytic solutions to the equations of motion of a gliding vehicle. We will make such assumptions in this analysis as well. The major assumptions are

1. The atmospheric density decreases exponentially with altitude
2. The acceleration due to gravity does not significantly change with altitude in the flight regime of interest
3. The flight-path angle does not change very much with time and is almost zero
4. The bank angle is constant

It may be argued that the third assumption may not apply in this analysis because the vehicle weight decreases with time along the powered-flight trajectory, resulting in increased altitude when the lift coefficient is constant. As will be shown later, the altitude does increase as propellant is expended, but the altitude change during flight is small, and consequently, the change in flight-path angle is very small.

EQUATIONS OF MOTION

The simplified equations of motion for powered and nearly horizontal flight are similar to those that describe gliding flight, with the effects of added thrust terms. Since the engine is canted downward with respect to the local flight path, thrust terms are present in the three directions of motion.

The equation of motion along the flight path is

$$\frac{1}{g}\frac{du}{dt} = \frac{1}{2g}\frac{du^2}{ds} = \frac{T(\cos\delta) - D}{W} \qquad (1)$$

where u is velocity, t is time, g is acceleration due to gravity, s is path length, T is thrust, δ is the cant angle of the engine with

respect to the flight path, D is the drag force, and W is the weight. The geometry is shown in Fig. 2.

The equation of motion in the vertical direction is

$$\frac{L + T(\sin \delta)}{W} \cos \varphi = 1 - \left(u^2/u_o^2\right) \tag{2}$$

where L is the lift force, φ is the bank angle, and u_o is the local circular satellite velocity at the cruise altitude.

Sideways motions for small distances away from the initial trajectory plane are described by

$$\frac{u}{g}\frac{d\omega}{dt} = \frac{L + T(\sin \delta)}{W} \sin \varphi \tag{3}$$

where ω is the heading change. Equation (3) assumes that lateral excursions away from the original path will be small. Generally, this assumption holds for steep bank angles or a low value of lift-to-drag ratio (L/D) at small bank angles. In this analysis we will be concerned with steeply banked vehicle attitudes ($\varphi \geq 60$ deg). The geometry is shown in Fig. 3. Finally, the change in weight due to expenditure of propellant is given by

$$\frac{dW}{dt} = - \frac{T}{I_{sp}} \tag{4}$$

where I_{sp} is the specific impulse of the propellant combination used in the engine.

CRUISE TRAJECTORIES

In considering the type of solutions sought here, we will make the component of thrust along the velocity vector equal to the drag force, so that a constant velocity will be maintained during powered turning flight. We introduce the term "thrust fraction," which is

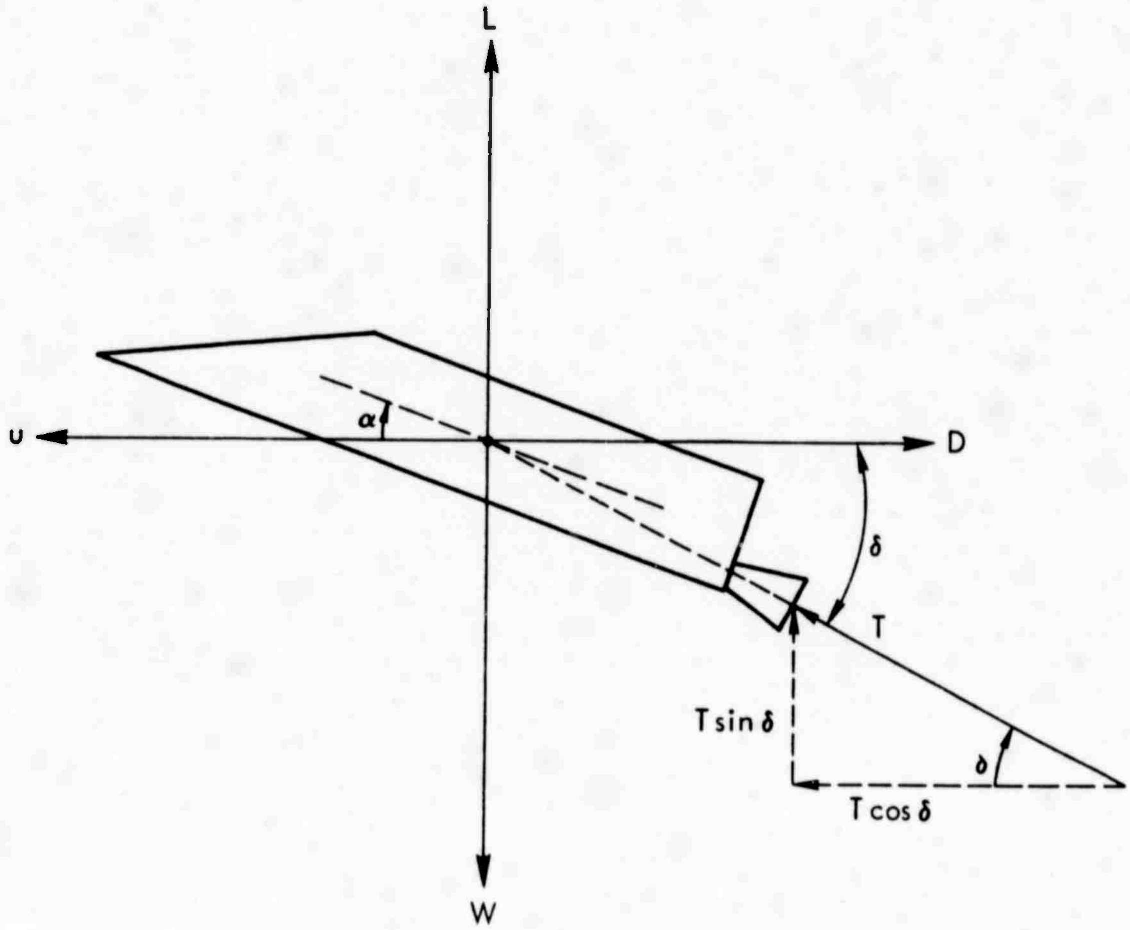

Fig. 2—Vehicle coordinate system

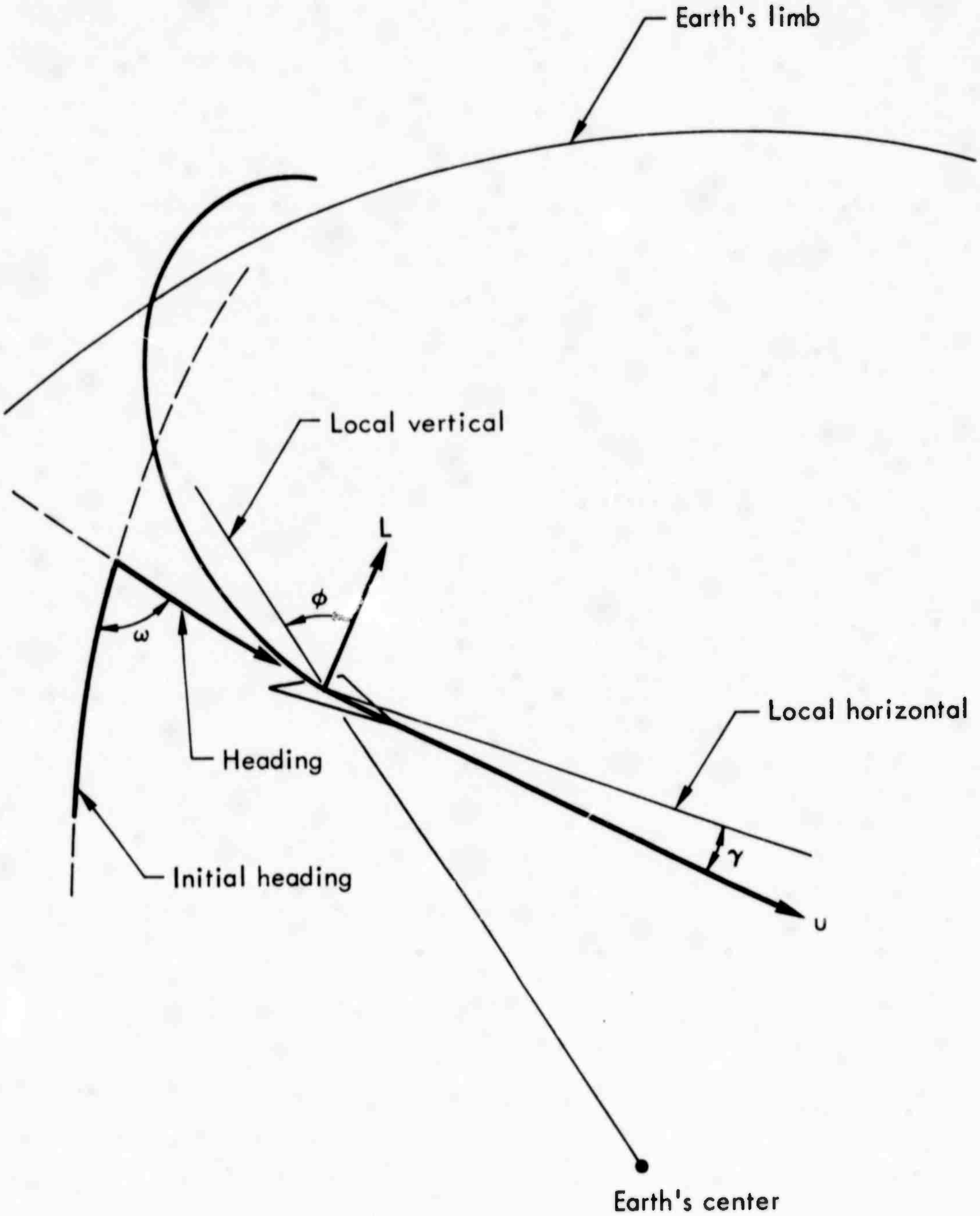

Fig. 3—Flight-trajectory coordinates

merely a coefficient expressing the thrust force as some fraction of the dynamic pressure on the flight path. Thus

$$T = C_T q$$

where q is the dynamic pressure, and C_T is the thrust fraction. We assume that C_T is a constant.

Further, it will be convenient to specify the engine cant angle at this time. Under the above assumptions, there is a best value for the cant of the engine for each value of vehicle L/D. We will use this cant angle, specified as

$$\tan \delta = D/L = C_D/C_L$$

where C_D and C_L are the drag and lift coefficients at the flight altitude of the vehicle. A brief analysis showing that this cant angle will minimize fuel expenditure during cruise is presented in the Appendix.

Combining the two statements given above with the constant-velocity condition, we find that one of the terms in the two equations of motion can be expressed more conveniently without the additional cant-angle notation:

$$L + T(\sin \delta) = qC_L A(1 + D^2/L^2)$$

where A is the aerodynamic reference area of the flight vehicle.

Solving for the aerodynamic pressure from Eq. (2),

$$q = \frac{\left[1 - \left(u^2/u_o^2\right)\right](W/C_L A)}{(1 + D^2/L^2)\cos \varphi} \qquad (5)$$

From this expression, the aerodynamic pressure is seen to be directly proportional to the vehicle weight. Thus the aerodynamic pressure will decrease as propellant is expended during the turn.

From Eq. (4) and the expression for dynamic pressure, the rate of weight change is given:

$$\frac{dW}{dt} = - \frac{C_T\left[1 - \left(u^2/u_o^2\right)\right](W/C_L A)}{I_{sp}(1 + D^2/L^2)\cos\varphi}$$

Integrating, the vehicle weight is found as a function of time, assuming constant I_{sp} and L/D:

$$\frac{W}{W_i} = \exp\left\{ - \frac{(C_T/C_L A)\left[1 - \left(u^2/u_o^2\right)\right]}{I_{sp}\cos\varphi(1 + D^2/L^2)} \right\} \tag{6}$$

Substituting the expression for dynamic pressure into Eq. (3), a simple equation describing sideways motion is generated:

$$\frac{u}{g}\frac{d\omega}{dt} = \left[1 - \left(u^2/u^2\right)\right]\tan\varphi$$

Integrating, the heading change is a linear function of time:

$$\omega = \frac{g\left[1 - \left(u^2/u_o^2\right)\right]\tan\varphi}{u}\, t \tag{7a}$$

Solving for time of flight:

$$t = \frac{u\omega}{g\left[1 - \left(u^2/u_o^2\right)\right]\tan\varphi} \tag{7b}$$

Substituting into Eq. (6), weight is now expressed as a function of heading change and cruise velocity:

$$\frac{W}{W_i} = \exp\left\{ - \frac{(C_T/C_L A)u\omega}{gI_{sp}\sin\varphi(1 + D^2/L^2)} \right\} \tag{8a}$$

Recalling the previous relations among thrust, drag, lift, and engine cant angle, Eq. (8a) may be written simply as

$$\frac{W}{W_i} = \exp\left\{ - \frac{u\omega}{gI_{sp} \sin \varphi \sqrt{(L^2/D^2) + 1}} \right\} \tag{8b}$$

This result is the counterpart of the classic rocket equation, but drag losses have been taken into account in analytic form, and the product $u\omega$ is viewed as an equivalent velocity increment.

If the weight of the fuel is to be made small, then several methods are suggested by the weight equation just developed. For a fixed plane change, a high specific impulse is beneficial. If the turn is made at high velocities (approaching orbital speed), then weight loss is minimized by using very large bank angles. Further, a high L/D would also appear to improve performance. Examples of calculations are shown in Fig. 4.

The velocity-altitude profile of the trajectory is found by assuming that the atmospheric density is an exponential function of altitude,

$$q = \frac{1}{2} \rho_o u^2 e^{-\beta h}$$

where ρ_o is the sea-level value of atmospheric density (slugs/ft^3), h is altitude, and β is the inverse scale height ($\beta \cong 1/24{,}000$ ft). Solving for altitude in Eq. (5),

$$\beta h = \log \frac{\rho_o g R_o}{2} - \log \left[\left(u_o^2/u^2 \right) - 1 \right] + \log \cos \varphi$$

$$+ \log(1 + D^2/L^2) - \log \frac{W_i}{C_L A} + \log \frac{W_i}{W} \tag{9}$$

The result is similar to that of previous analyses of gliding flight but contains some additional terms.

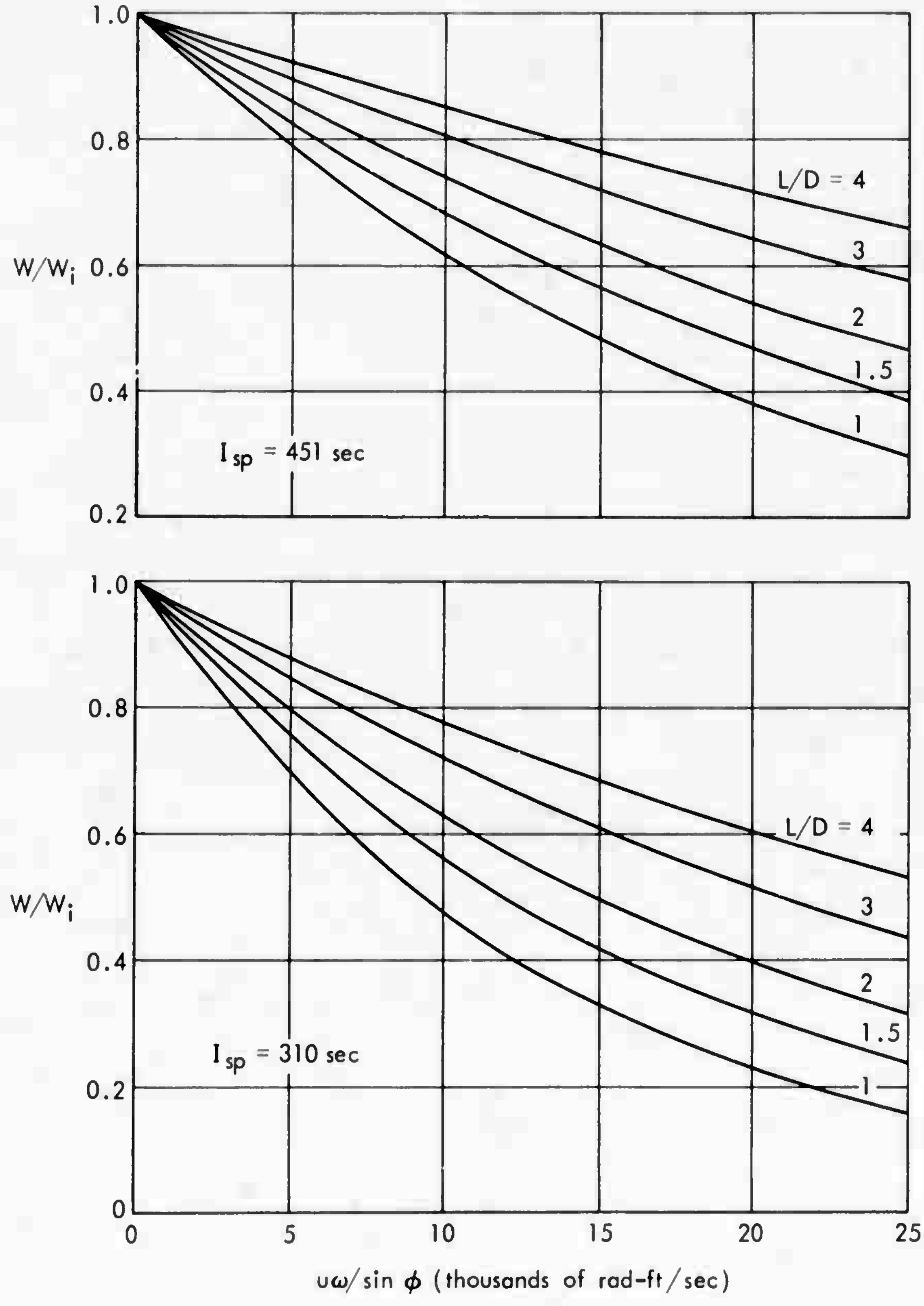

Fig. 4—Weight remaining after cruise

As noted earlier, the altitude during powered flight will increase because propellant is expended to provide thrust. Thus, the flight-path angle will have some small positive value. So long as only a portion (50 percent or less) of the vehicle weight is devoted to propellant, the initial assumptions concerning flight-path angle will be nearly correct and should not introduce large errors into the analytic results.

A generalized plot of altitude as a function of velocity is shown in Fig. 5. Example corrections to this plot to meet other flight parameters are shown in Fig. 6; similar curves may be constructed of the trajectory shape of interest in particular cases. A specific example will be included in Section III.

ASCENT TRAJECTORIES

The performance for ascending from the atmosphere to orbit can be derived from the basic equations of motion. For this phase of flight, the vehicle accelerates and the velocity increases with time. We assume that the thrust-to-drag ratio is constant and greater than unity and that the cant angle δ is also a constant.

Under these conditions, the weight of the vehicle is found to be

$$\frac{W}{W_i} = \exp\left[-\frac{u - u_i}{gI_{sp}(\cos\delta - D/T)}\right] \tag{10}$$

Equation (10) is a result of combining Eqs. (1) and (4) and integrating, under the assumptions stated above.

During the ascent back to orbit, it is possible to make an additional heading change over that achieved during cruise. Combining Eqs. (1) and (3), the amount of heading change is inversely proportional to the forward acceleration:

$$\omega = \frac{L + T\sin\delta}{D(k - 1)}\sin\varphi \log\frac{u}{u_i} \tag{11}$$

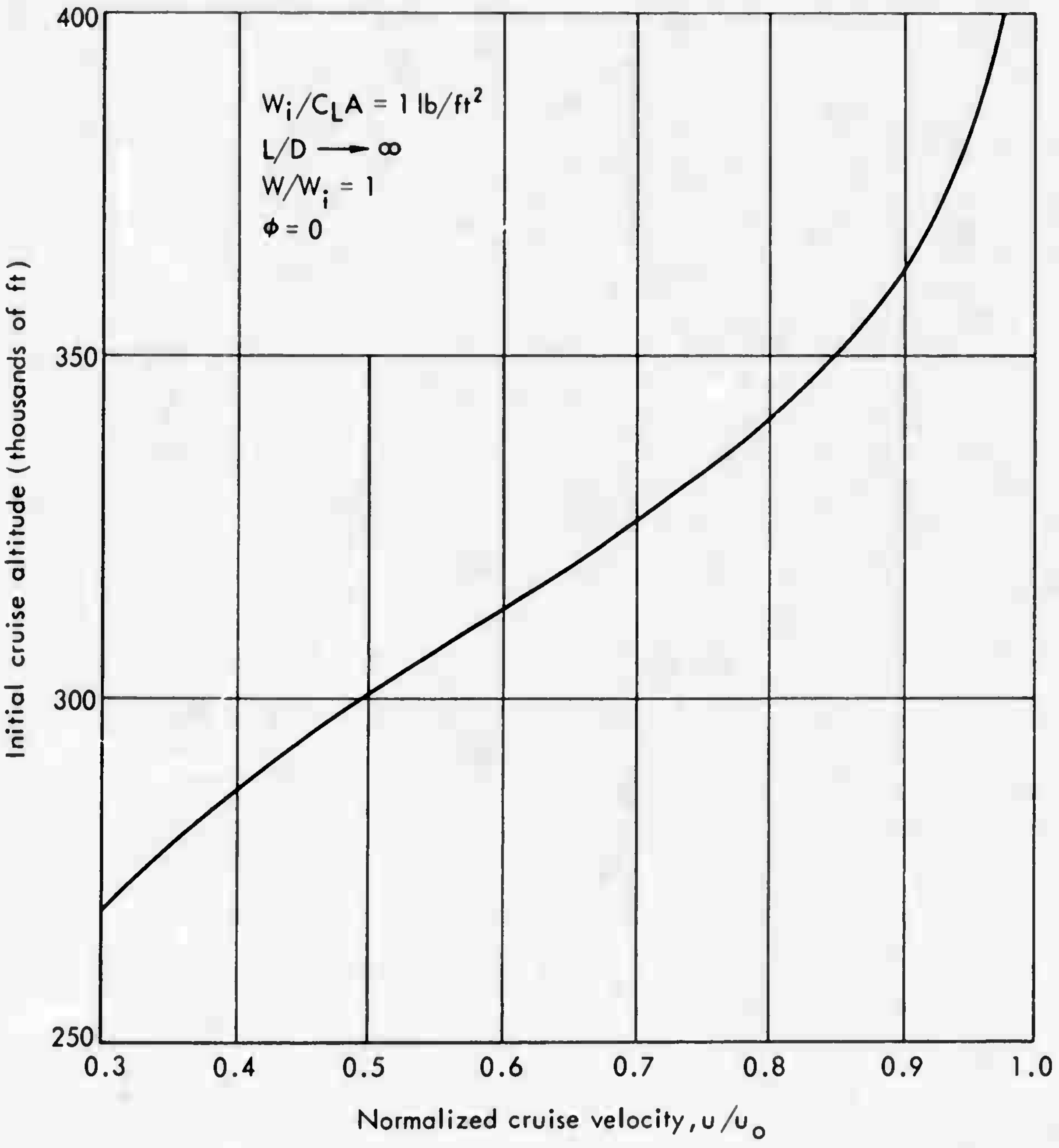

Fig. 5—Glide trajectory to cruise initiation

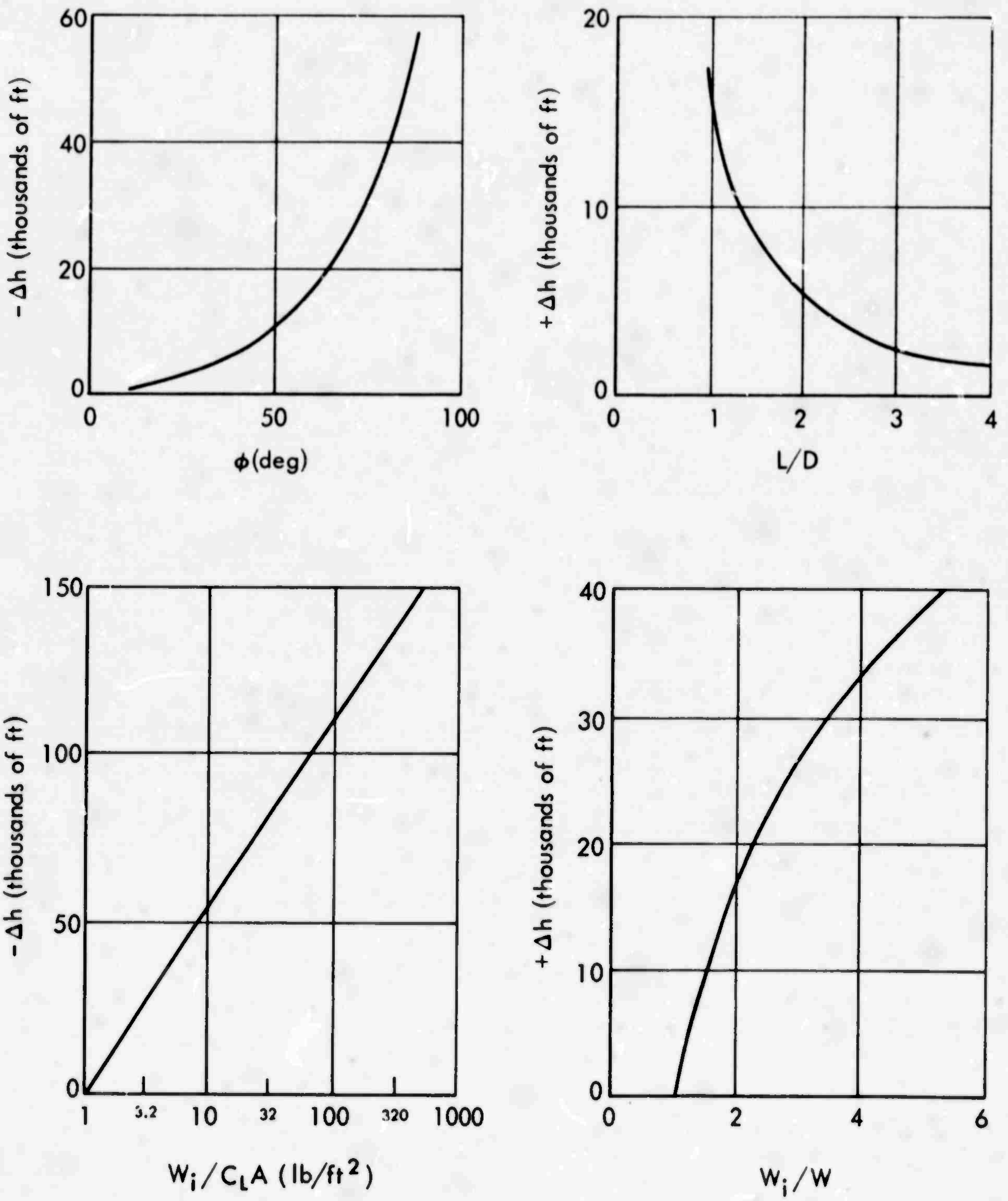

Fig. 6—Altitude corrections for glide and cruise

where $k = T(\cos\delta)/D$, the ratio of the forward-pushing thrust force to the drag force. If the cruise velocity is near orbital speed, the amount of heading change achieved during the accelerating portion of flight will be small because of the small velocity ratio.

The altitude-velocity profile is found by solving for dynamic pressure and then altitude:

$$\beta h = \log \frac{\rho_o u_o^2}{2} + \log \frac{u^2/u_o^2}{1 - \left(u^2/u_o^2\right)} + \log \cos\varphi - \log \frac{W_i}{C_L A}$$

$$- \log \frac{W}{W_i} + \log \left(1 + \frac{k \tan\delta}{L/D}\right) \tag{12}$$

This expression is almost identical to Eq. (9), with the exception of the last term. In essence, the vehicle would fly along the same trajectory as it would in entry, except at a higher altitude because of propellant expenditure and thrust-aided lift (the last two terms). These terms are plotted in Fig. 7.

The path length during ascent may be found by letting $\delta = 0$ and noting that $T = kD$. Then the two equations used to solve for path length are

$$\frac{1}{2g} \frac{du^2}{ds} = (k - 1) \frac{D}{W}$$

and

$$q \frac{C_L A}{W} \cos\varphi = 1 - \left(u^2/u_o^2\right)$$

Solving for the dynamic pressure in the latter equation and substituting into the former, the result of integration yields path length:

$$s = \frac{R_o}{2(k - 1)} \left(\frac{L}{D} \cos\varphi\right) \log \frac{1 - \left(u_i^2/u_o^2\right)}{1 - \left(u^2/u_o^2\right)} \tag{13}$$

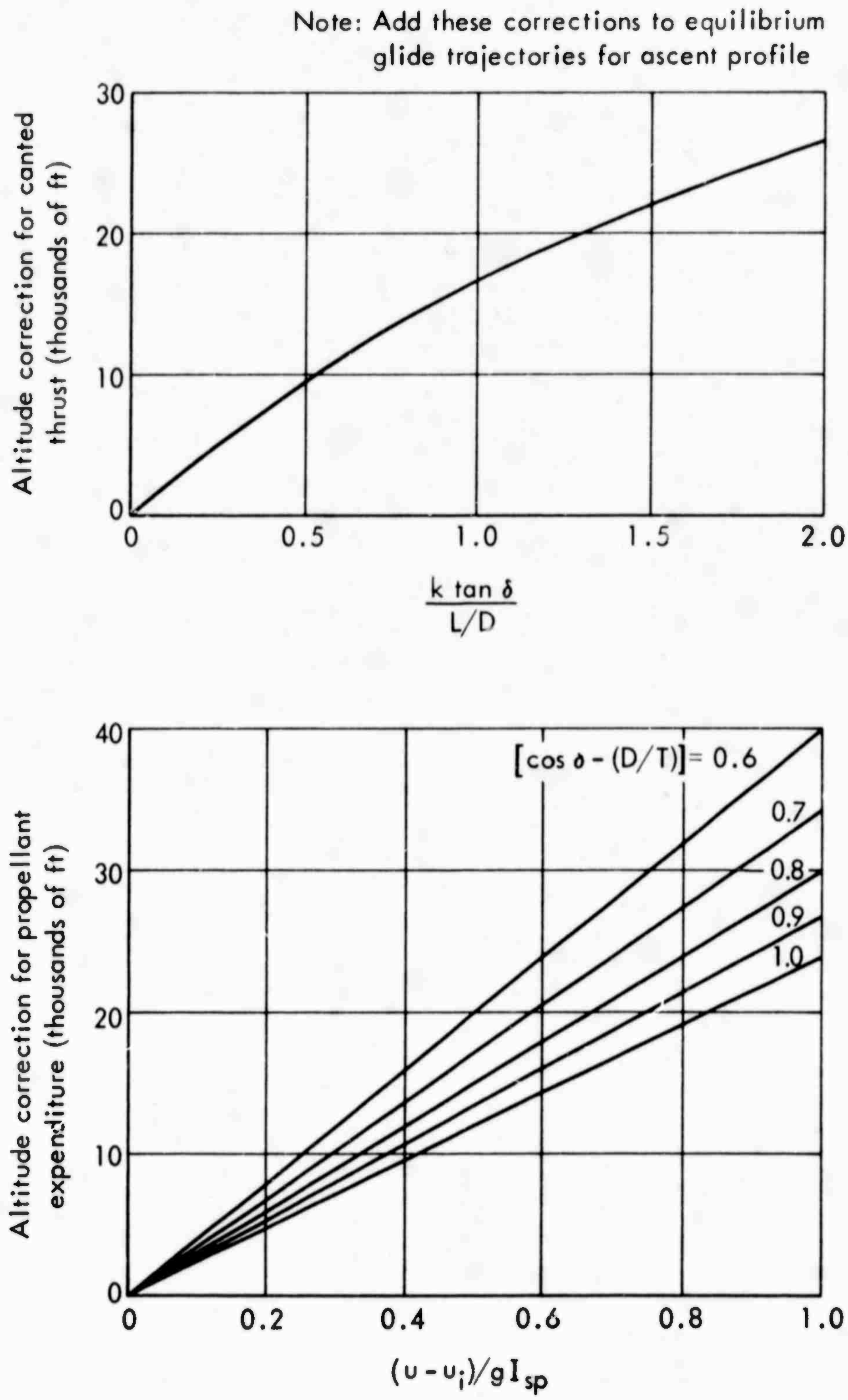

Fig. 7—Corrections for ascent trajectories

This result is similar in many ways to the range equation for glide vehicles. The major difference is that Eq. (13) includes the effect of forward acceleration by the inclusion of the term (k - 1).

If it is of interest to find the shape of the ground track during ascent, the approach is similar to that developed earlier.[6] The downrange and side-range distances for constant-bank-angle control are found by noting that

$$\frac{dx}{du} = \frac{ds}{du} \cos \omega$$

$$\frac{dy}{du} = \frac{ds}{du} \sin \omega$$

where x is the distance traveled in the original direction of motion at the start of the ascent to orbit, and y is the distance traveled perpendicular to the original direction of motion. Differentiating the range equation with respect to velocity,

$$g \frac{ds}{du} = \frac{L/D}{k - 1} \frac{\cos \omega}{2} \frac{u}{1 - \left(u^2/u_o^2\right)} \tag{14}$$

which is nearly the same as the result derived in Ref. 5, except that the term (k - 1) now is divided into L/D. Thus, we can calculate an effective L/D and use the results of the earlier, somewhat complicated integration derived for glide vehicles. For example, if L/D = 3 and k = 4, the effective L/D during ascent to orbit would be unity, and the previous results of Ref. 5 for L/D = 1 would apply.

HEATING TRENDS

One proposed advantage of a constant-velocity cruise in a synergetic maneuver is a reduction in heating, as well as in drag losses.

The heating rate encountered by a vehicle flying in the upper atmosphere is generally proportional to the dynamic pressure and velocity

of the vehicle.[7] The equation for calculating the approximate heating rate at the stagnation point under cold-wall conditions is

$$\frac{dH_s}{dt} = \frac{C}{\sqrt{r_n}}\left(\frac{\rho}{\rho_o}\right)^{\frac{1}{2}}\left(\frac{u}{u_o}\right)^3 \tag{15}$$

where C is a constant with a value of 16,000 to 20,000 Btu-ft$^{3/2}$/sec for air, and r_n is the nose radius. For the cruise portion of flight, this equation becomes

$$\frac{dH_s}{dt} = \frac{C}{\sqrt{r_n}}\, f(u)\sqrt{\frac{2(W/C_L A)}{u_o^2\rho_o(1 + D^2/L^2)\cos\varphi}} \tag{16}$$

where f(u) is the cruise-velocity function defined by

$$f(u) = \frac{u^2}{u_o^2}\sqrt{1 - \left(u^2/u_o^2\right)}$$

Examination of this function (see Fig. 8) shows that heating rates can be reduced substantially for any given vehicle by cruising at velocities at or above $0.90u_o$. The maximum heating rates would occur at cruise velocities of about $0.8u_o$ (these velocities should, of course, be avoided if maximum heating rates are to be avoided). Equation (16) also shows the effects of thrust, bank angle, and expenditure of propellant. Although heating rates increase with increasing bank angles ($0 < \varphi < \pi/2$), this increase may be offset by the thrust and decreasing weight.

A more specific heating model may be generated to indicate the amount of temperature decrease to be expected from the use of a cruise phase as compared to that which occurs when the vehicle is allowed to glide until the desired heading change is achieved. For this comparison, we have constructed a model giving the temperature of a point 1 ft aft of the leading edge of a flat plate at a 10-deg angle of attack.

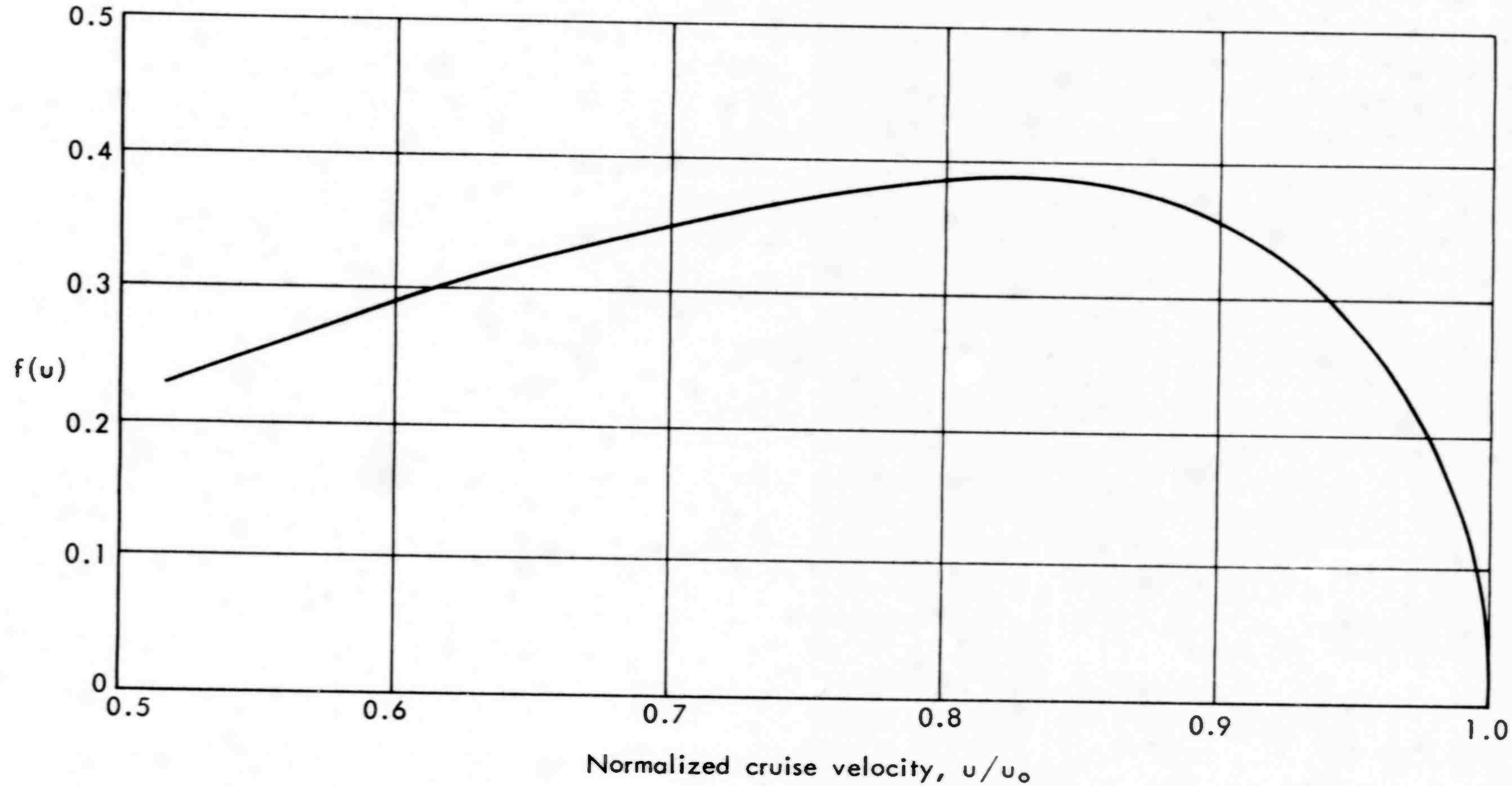

Fig. 8 — Velocity function of heating rate during cruise

Briefly, the model is based on the reference-enthalpy method of Eckert.[8] The pressure, temperature or enthalpy, and velocity used for the calculations are those downstream of the oblique shock at the outer edge of the boundary layer. Transport properties of the air in the boundary layer are evaluated at the reference enthalpy described by Eckert.

Equilibrium surface temperatures are computed by setting the convective heat flux equal to the heat flux radiated from the surface, assuming no heat transfer to the interior. In principle, this is not possible as long as the interior structure is at a temperature below that of the exterior surface, but for most entry vehicles the subsurface materials such as insulation have a relatively small heat conductivity, and the interior heat flow may be nearly negligible. Thus, the aerodynamic convective-heat input to the surface is assumed to be exactly balanced by the radiative-heat output from the surface.

Calculations of equilibrium surface temperature based on this model for turbulent flow are presented in Fig. 9. Similar calculations could be made for laminar flow as well but are not carried out here. One would expect generally lower temperatures under the assumptions of laminar flow. An example is given in Section III which involves some of the data presented here.

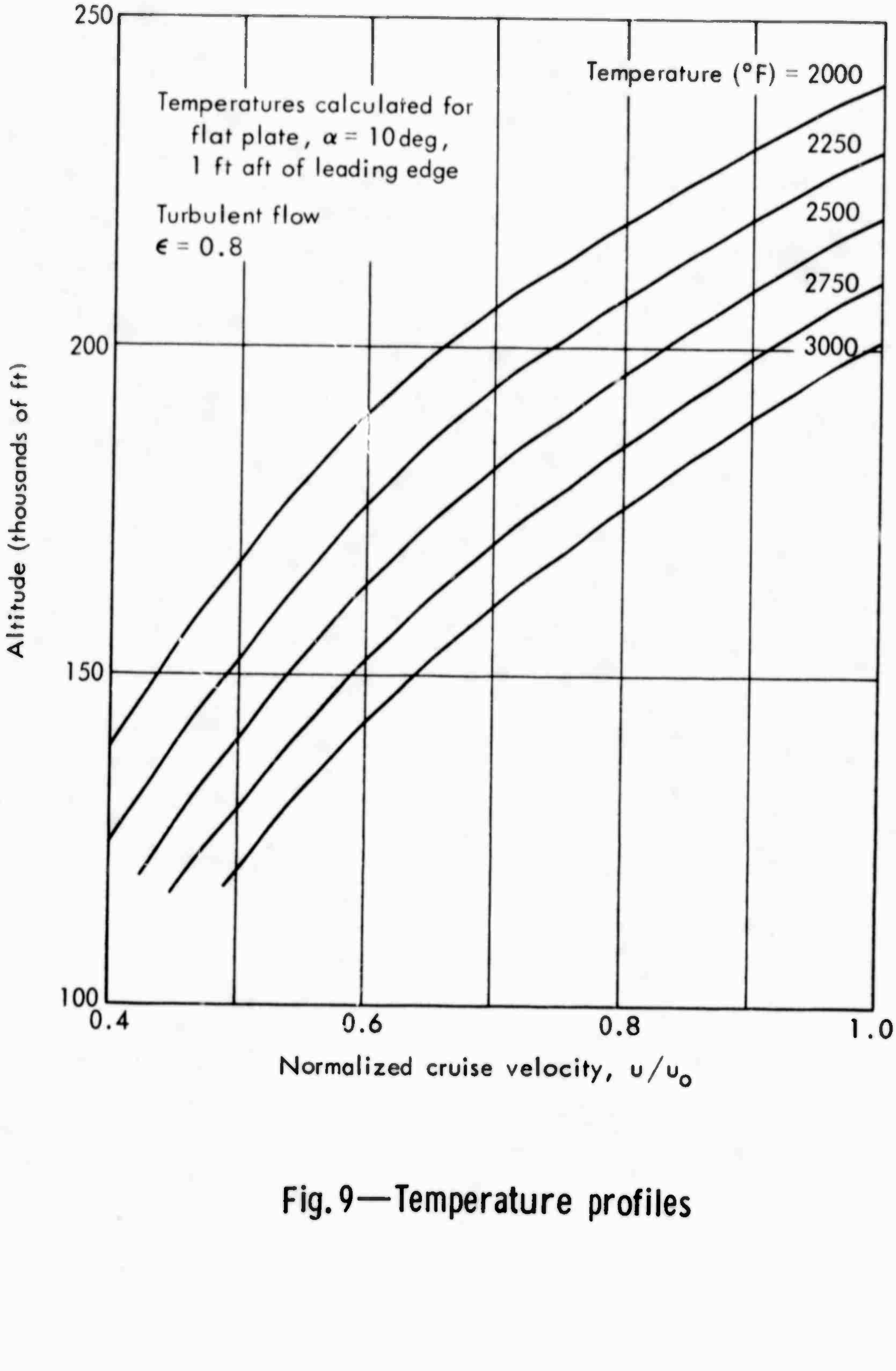

Fig. 9—Temperature profiles

III. AN EXAMPLE CALCULATION

From the previous discussion, it is possible to construct a number of examples of synergetic plane changes with a cruise-flight phase. This section presents such calculated results for a vehicle with an L/D of 3 and a propulsion-system specific impulse of 451 sec, which is representative of systems using liquid hydrogen and liquid oxygen as propellants. The engine cant angle is considered to be null during ascent, and no credit is taken for turning during this portion of the flight regime.

We assume that the spacecraft is banked at 75 deg during the descent to the atmosphere and that it enters along an equilibrium glide trajectory. We also assume that $W_i/C_L A = 200$ lb/ft^2, similar to past vehicle designs. The vehicle would enter on the trajectory shown in Fig. 10. If permitted to slow down, its heading would change as indicated by the notation along the solid curve. However, if thrust is applied when the vehicle reaches a velocity $u = 0.9u_0$, the cruise phase would be started. The dashed line shows the resulting trajectory; the heading change during cruise is indicated, along with that achieved during the initial slow-down phase. The result, discussed in general terms earlier, is that the altitude increases during cruise, the vehicle does not descend as far into the atmosphere as it would in a pure gliding turn, and normal load factors are decreased by cruising.

If the ascent takes place along the type of trajectory described earlier, the amount of propellants for both types of turning flights can be calculated. For $\varphi = 0$ and $\delta = 0$, during ascent to a velocity of 26,000 ft/sec, it appears that slightly better performance is achieved when a cruise phase is used: Calculated results are shown in Fig. 11. The thrust of the engine was assumed to be the same for all plane changes and was fixed so that the thrust-to-drag ratio, T/D, is 10 for a heading change of 17.5 deg. If the vehicle were permitted to glide farther into the atmosphere, increased drag losses were incurred because of the reduced T/D.

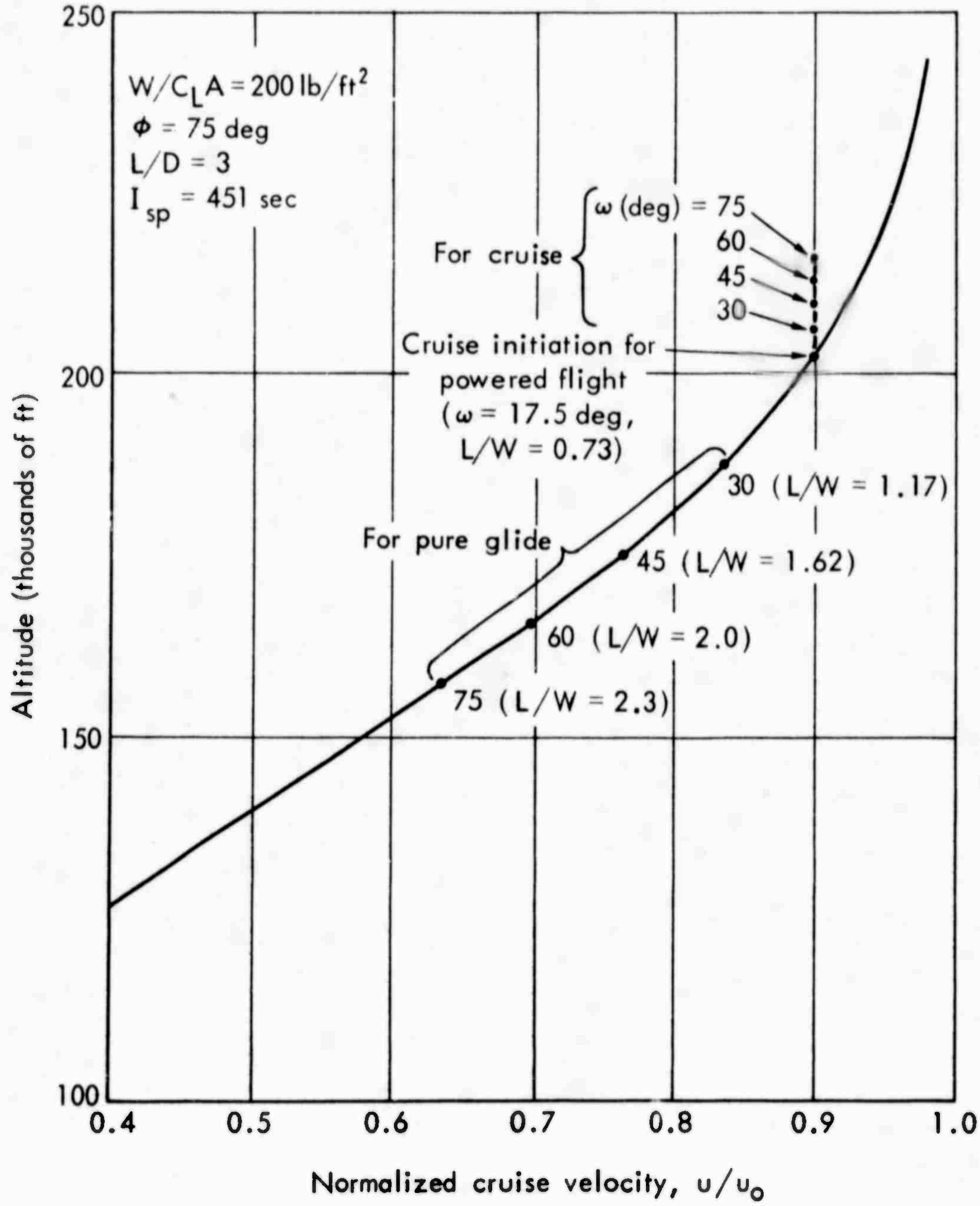

Fig. 10—Trajectory comparison: powered versus unpowered flight

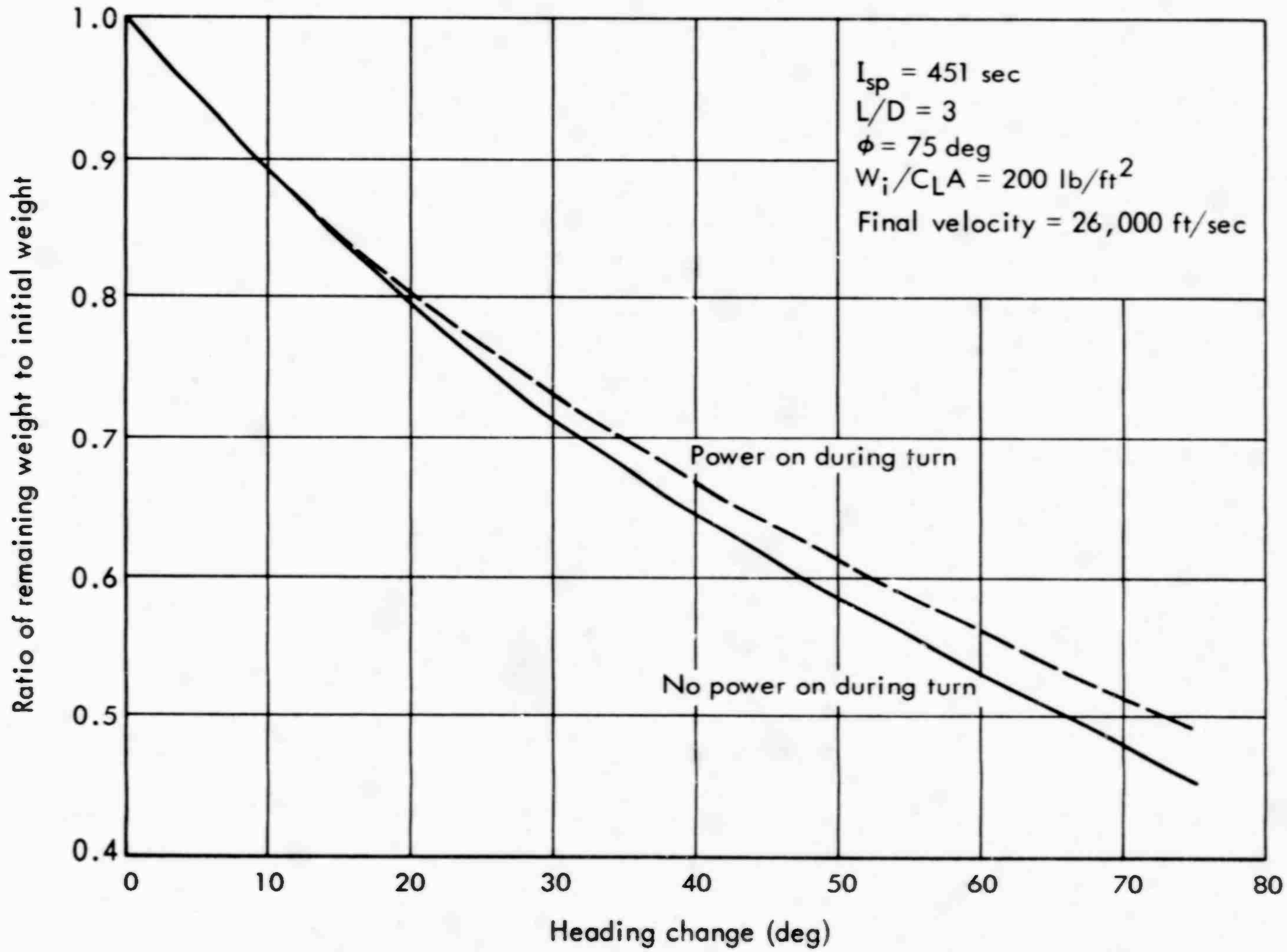

Fig. 11—Example of performance comparison

This result might be expected, since the average value of drag would be lowered if the vehicle remained at high altitudes. Whether or not this advantage would result in a more detailed analysis remains to be seen. In this analysis, the L/D is assumed constant; however, in reality, it is a function of altitude and generally decreases with increasing altitude. Thus, there would appear to be a cruise altitude that is high enough to result in performance improvement above a pure glide mode, but not so high as to degrade performance to the point where the propellant expended is about the same as that expended if the plane change were accomplished in orbit by propulsive means alone.

Finally, we shall examine the temperature of the structure to determine if use of a cruise phase is beneficial from a heating standpoint. From an earlier example in the discussion of heating (pp. 16 - 18), which is reproduced in Fig. 12, it is seen that some decrease in structural temperature appears likely if a cruise phase is employed, as compared to a pure glide trajectory. In this example, a reduction has been achieved because the cruise velocity was selected to be $0.9u_o$. Higher cruise velocities would appear to result in lower temperatures because of the shape of the altitude-velocity profile at near-orbital speeds. As shown earlier, substantial heating-rate reductions are possible at higher cruise velocities. In a more detailed analysis, however, one would consider the lower L/D and its effect on performance at the higher altitudes and velocities. Thus, while heating rates or temperatures might be substantially reduced, a penalty will probably be encountered in the form of more propellant expenditure. For example, in this case, use of propulsion has decreased the maximum temperature by more than 150^oF.

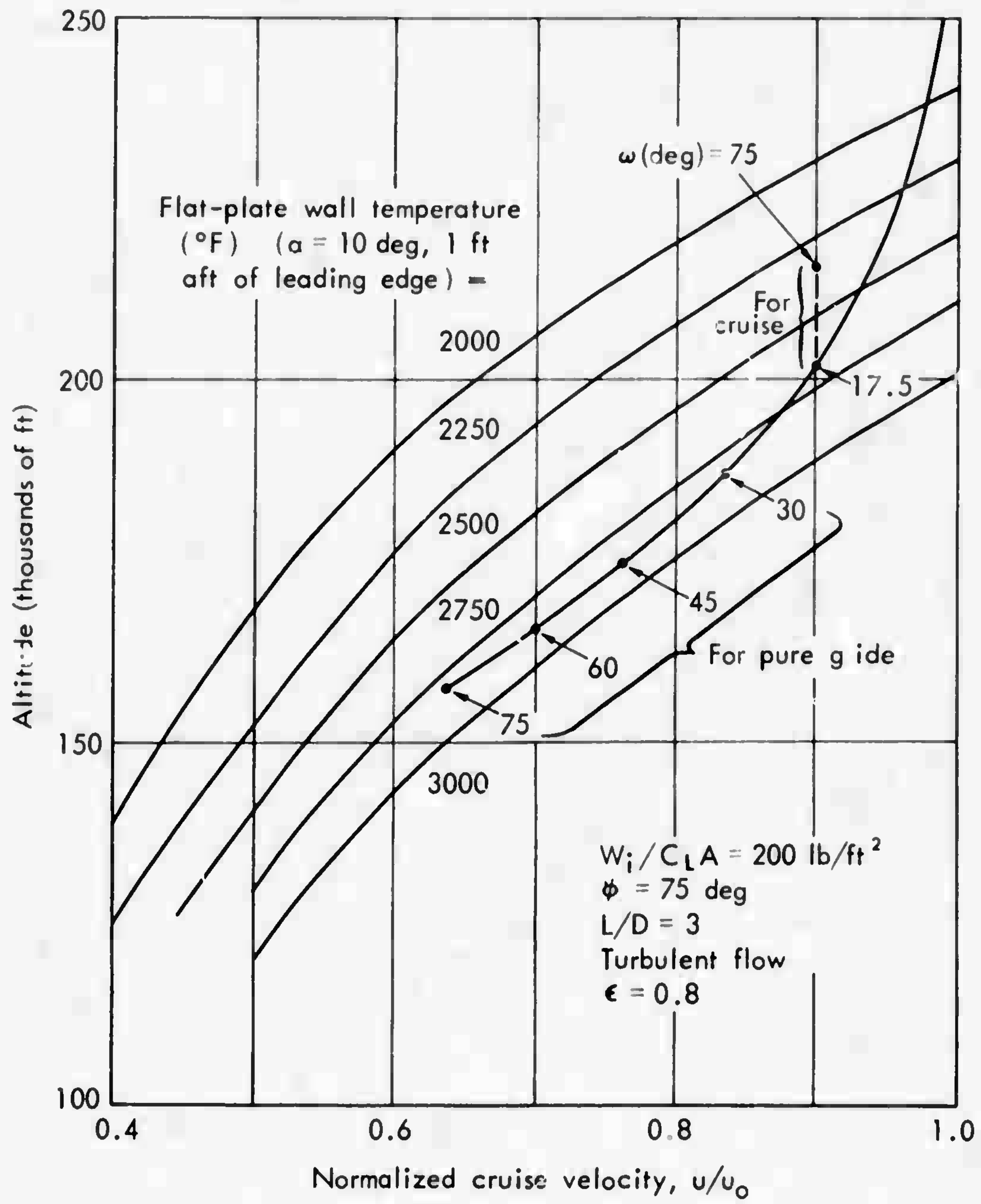

Fig. 12—Temperature profiles and trajectories

IV. CONCLUSIONS

In this Memorandum, the use of a cruise phase of flight has been suggested as one approach to reducing heating loads during synergetic plane changing. Hand solutions were obtained by making the usual linearizing assumptions about the equations of motion. It was found that structural temperatures of lifting vehicles can be reduced by utilizing power during the turning phase of flight, compared to the temperatures encountered along trajectories where turning is accomplished during a pure glide phase. The use of propulsion during a turn may permit the use of highly banked flight attitudes which tend to minimize energy expenditures in such maneuvers. Another benefit of cruising is a reduced normal acceleration.

This analysis did not include variations of L/D with altitude and velocity. In more detailed analyses, such variations would be important and could lead to determining a "preferred" cruise altitude which would alleviate heating problems but not cause large performance losses due to a lowered L/D. It was found that heating could be reduced and that increases in cruise velocity could be used to effect decreased temperatures. If, for example, the cruise velocity were 95 percent of orbital velocity, then the maximum temperature could be reduced by about $500^{\circ}F$ in the sample calculations presented (see Fig. 12). Thus, it appears that there is a potential for using propulsion to limit thermal problems during synergetic plane changes.

Appendix

ENGINE-CANT-ANGLE SELECTION

In this appendix we will analyze the equations of motion to determine a preferred engine cant angle, δ, that minimizes fuel consumption. This value of engine cant angle has been used in analyzing the cruise phase of flight in the body of this Memorandum.

For the cruise phase of flight we shall derive the weight history without specifying the engine cant angle. Rearranging Eq. (2), the aerodynamic pressure may be written as

$$q = \frac{(W/C_L A)\left[1 - \left(u^2/u_o^2\right)\right]}{[1 + (T/L)\sin\delta]\cos\varphi}$$

where the ratio of thrust to lift is constant. Substituting into Eq. (4) after letting $T = C_T q$,

$$\log\frac{W}{W_i} = \frac{\left[1 - \left(u^2/u_o^2\right)\right] t}{I_{sp}\cos\varphi[(L/T) + \sin\delta]}$$

Solving for time of flight from Eq. (3) and substituting into the above result,

$$\log\frac{W}{W_i} = \frac{u\omega}{gI_{sp}\sin\varphi[(L/T) + \sin\delta]}$$

To minimize fuel expenditure when u, ω, I_{sp}, and φ are constant, we must maximize the term $[(L/T) + \sin\delta]$.

Let

$$f(\delta) = (L/T) + \sin\delta$$

but since $T = D/\cos\delta$

$$f(\delta) = (L/D)\cos\delta + \sin\delta$$

Solving for the maximum, we find that it occurs when

$$\tan\delta = D/L$$

This result is used in the body of this Memorandum.

With regard to the best value for engine cant angle during ascent to orbit, it appears that imposing the condition $\delta = 0$ will maximize the remaining weight after the engine is shut down. This result is readily seen by examination of Eq. (10).

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10. ABSTRACT

An investigation of the interactions of thrust and lift on equilibrium reentry trajectories to determine if the use of thrust during a turn will reduce the heat input to a winged spacecraft. In the maneuver considered in this study, propulsion is used shortly after the vehicle enters the atmosphere to cancel drag and provide additional lift, so that much of the heading change will occur during the cruise phase at constant vehicle velocity. Engine thrust is increased at the end of cruise to accelerate the vehicle into a new orbit, thus decreasing heating rates and vehicle temperatures. A variable-thrust engine is assumed. The study shows that by using propulsion during the turning phase of flight, the st.uctural temperatures of lifting vehicles can be lowered by about 500 degrees F, compared with the temperatures encountered when the turn is made during a pure glide phase. If the turn is made at high velocity, fuel loss is minimized by using very large bank angles. No losses in vehicle performance were noted, and normal load factors were decreased.

11. KEY WORDS

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